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Strength of Materials (GATE Aerospace and GATE Mechanical) by Mr Dinesh Kumar

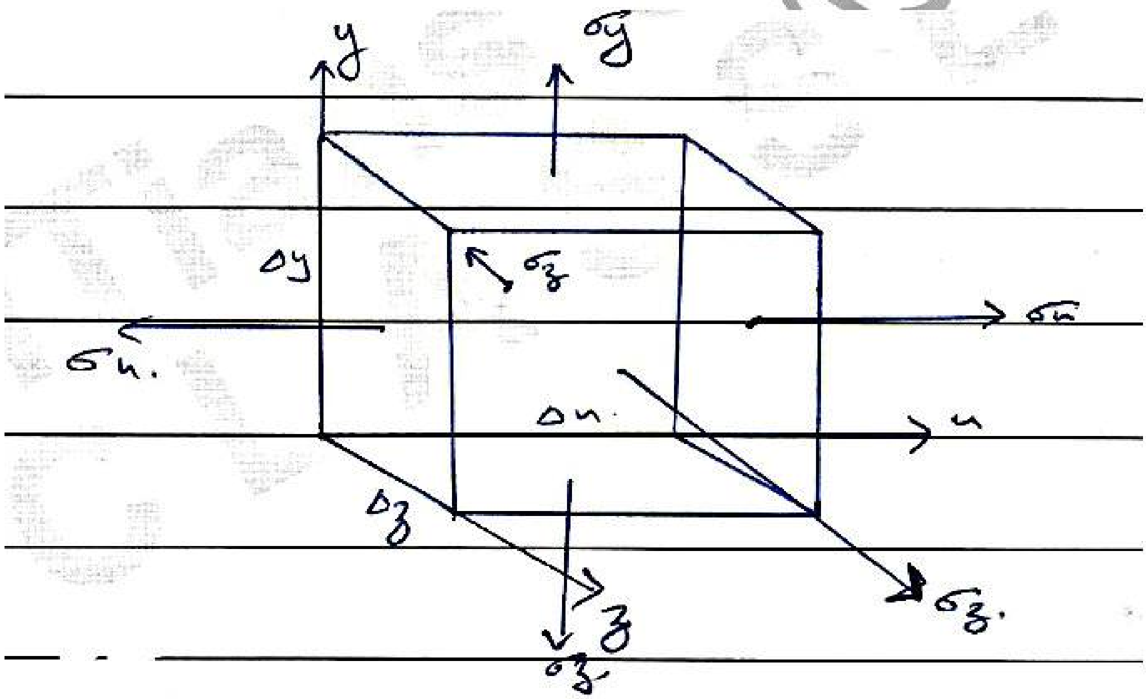
Volumetric Strain or Dilatations

- Volumetric strain occurs only due to normal stress.
- Shear stress causes only distortion and has no change in volume.

Change in volume to original volume due applied normal stresses is known as volumetric strain.

$$e = \frac{change\ in\ volume}{origional\ volume} = \frac{\Delta V}{V}$$

Consider a triaxial stress system as shown in the figure



Final length of cuboids in x, y and z directions are given by

$$\Delta x' = (1+\epsilon_x)\ \Delta x$$

$$\Delta y' = (1+\epsilon_y)\ \Delta y$$

$$\Delta z' = (1+\epsilon_z)\,\Delta z$$

Final volume of cuboids

$$\Delta V' = \Delta x'.\Delta y'.\Delta z'$$

Initial volume of cuboids

$$V = \Delta x.\,\Delta y.\,\Delta z$$

Volumetric strain

$$e = \frac{V' - V}{V}$$

$$e = \frac{(1+\epsilon_x)\Delta x(1+\epsilon_y)\Delta y(1+\epsilon_z)\Delta z - \Delta x\Delta y\Delta z}{\Delta x\Delta y\Delta z}$$

$$e = \frac{(1+\epsilon_x)\Delta x(1+\epsilon_y)\Delta y(1+\epsilon_z)\Delta z - \Delta x\Delta y\Delta z}{\Delta x\Delta y\Delta z}$$

$$e = (1+\epsilon_x)(1+\epsilon_y)(1+\epsilon_z) - 1$$

Neglecting the higher order term of small quantities

$$\mathbf{e = 1 + \epsilon_x + \epsilon_y + \epsilon_z - 1}$$

$$\mathbf{e = \epsilon_x + \epsilon_y + \epsilon_z}$$

Substituting strain in terms of stress

$$e = \frac{(1-2\nu)}{E}(\sigma_x + \sigma_y + \sigma_z)$$

For hydrostatic loading (pressure),

$$\rightarrow \sigma_x = \sigma_y = \sigma_z = -P \text{ (if it is inwards)}$$

$$e = \frac{-3(1-2\nu)P}{E}$$

$$K = \frac{E}{3(1-2\nu)}$$

Here, K is known as Bulk modulus of elasticity and is measure of compressibility of solid

$$\therefore e = -\frac{P}{K}$$

Value of ν is limited between, $-1 \leq \nu \leq 0.5$

Upper limit is decided by Bulk modulus and lower limit is decided by Modulus of rigidity G

Problems

1.

The Poisson's ratio for a perfectly incompressible linear elastic material is

(A) 1 (B) 0.5 (C) 0 (D) infinity

Sol. B

Compressibility of a solid is measured by bulk modulus,

K = $\frac{E}{3(1-2\nu)}$

For perfectly linear elastic materials

k = ∞

So, $\frac{1}{0} = \frac{E}{3(1-2\nu)}$

ν = 0.5

2.

The value of Poisson's ratio at which the shear modulus of an isotropic material is equal to the bulk modulus is

(A) $\frac{1}{2}$

(B) $\frac{1}{4}$

(C) $\frac{1}{6}$

(D) $\frac{1}{8}$

Sol. D

We know

Modulus of rigidity or shear modulus, G = $\frac{E}{2(1+\nu)}$

Bulk modulus, K = $\frac{E}{3(1-2\nu)}$

Given, G = K

$\frac{E}{2(1+\nu)} = \frac{E}{3(1-2\nu)}$

3 - 6ν = 2 + 2ν

1 = 8ν

ν = 1/8

3.

A cube made of a linear elastic isotropic material is subjected to a uniform hydrostatic pressure of 100 N/mm^2. Under this load, the volume of the cube shrinks by 0.05%. The Young's modulus of the material, $E = 300\ GPa$. The Poisson's ratio of the material is _____.

Sol. 0.25

Given hydrostatic pressure P = 100 N/mm^2
Volume of cube is shrinking by 0.05%.
Volumetric strain e = - $\frac{0.05}{100}$ = - 0.0005
Young's modulus E = 300 GPa
We know,

$$e = \frac{-3(1-2\nu)}{E} \times p$$

$$-0.0005 = \frac{-3(1-2\nu)}{300\times1000} \times 100$$

$$0.5 = (1-2\nu)$$

$$\nu = 0.25$$

4.

A rod is subjected to a uni-axial load within linear elastic limit. When the change in the stress is 200 MPa, the change in the strain is 0.001. If the Poisson's ratio of the rod is 0.3, the modulus of rigidity (in GPa) is ______________

Sol. 76.92GPa

Given,

Change in the stress $\Delta\sigma$ = 200 MPa

Change in the strain $\Delta\epsilon$ = 0.001

Poisson's ratio ν = 0.3

We know,

Young's modulus E = $\frac{\Delta\sigma}{\Delta\epsilon} = \frac{200\times10^6}{0.001}$ = 200GPa

Modulus of rigidity G = $\frac{E}{2(1+\nu)} = \frac{200\times10^9}{2(1+0.3)}$ = 76.92GPa

5.

If the Poisson's ratio of an elastic material is 0.4, the ratio of modulus of rigidity to Young's modulus is _______

Sol. 0.357

Given Poisson's ratio ν = 0.4

We know G = $\frac{E}{2(1+\nu)}$

Ratio of $\frac{G}{E} = \frac{1}{2(1+\nu)} = \frac{1}{2.8}$ = 0.357

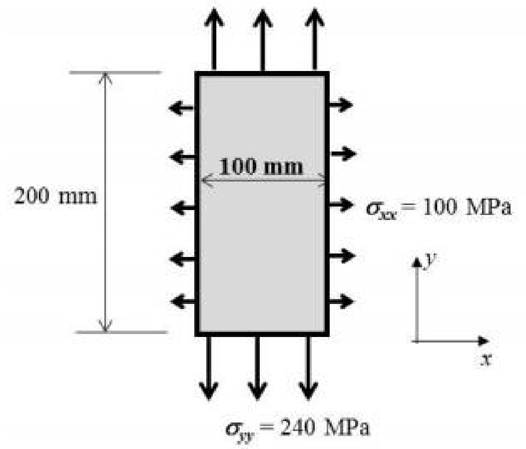
6.

The number of independent elastic constants required to define the stress-strain relationship for an isotropic elastic solid is _______

Sol. 2

7.

A thin plate with Young's modulus 210 GPa and Poisson's ratio 0.3 is loaded as shown in the figure. The change in length along the y-direction is __________ mm (*round off to 1 decimal place*).



Sol. 0.2 mm

Given E = 210 GPa, L_x = 100 mm, σ_{xx} = 100 MPa

ν = 0.3, L_y = 200 mm, σ_{yy} = 240 MPa

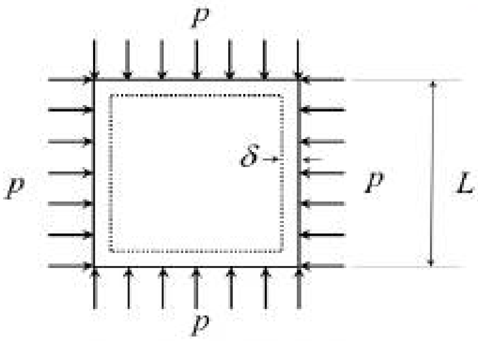
$$\epsilon_y = \frac{\sigma_{yy}}{E} - \nu\frac{\sigma_{xx}}{E}$$

$$\frac{\Delta L_y}{L_y} = \frac{1}{E}(\sigma_{yy} - \nu\sigma_{xx}) = \frac{1}{210\times1000}(240 - 0.3\times100) = 0.001$$

$$\Delta L_y = 0.001\times200 = 0.2 \text{ mm}$$

8.

A square plate of dimension $L \times L$ is subjected to a uniform pressure load $p = 250$ MPa on its edges as shown in the figure. Assume plane stress conditions. The Young's modulus $E = 200$ GPa.



The deformed shape is a square of dimension $L - 2\,\delta$. If $L = 2$ m and $\delta = 0.001$ m, the Poisson's ratio of the plate material is __________

Sol. 0.2

Given P = -250 MPa

E = 200 GPa

L' = L-2δ

L = 2 m, δ = 0.001 m

Here, $\epsilon_x = \epsilon_y = \frac{1}{E}(P - \nu P)$

$\frac{\Delta L'}{L} = \frac{L' - L}{L} = \frac{P}{E}(1 - \nu)$

$\frac{L - 2\delta - L}{L} = \frac{P}{E}(1 - \nu)$

$\frac{-2 \times 0.001}{2} = \frac{-250}{200 \times 1000}(1 - \nu)$

$(1 - \nu) = \frac{200}{250} = 0.8$

$\nu = 0.2$

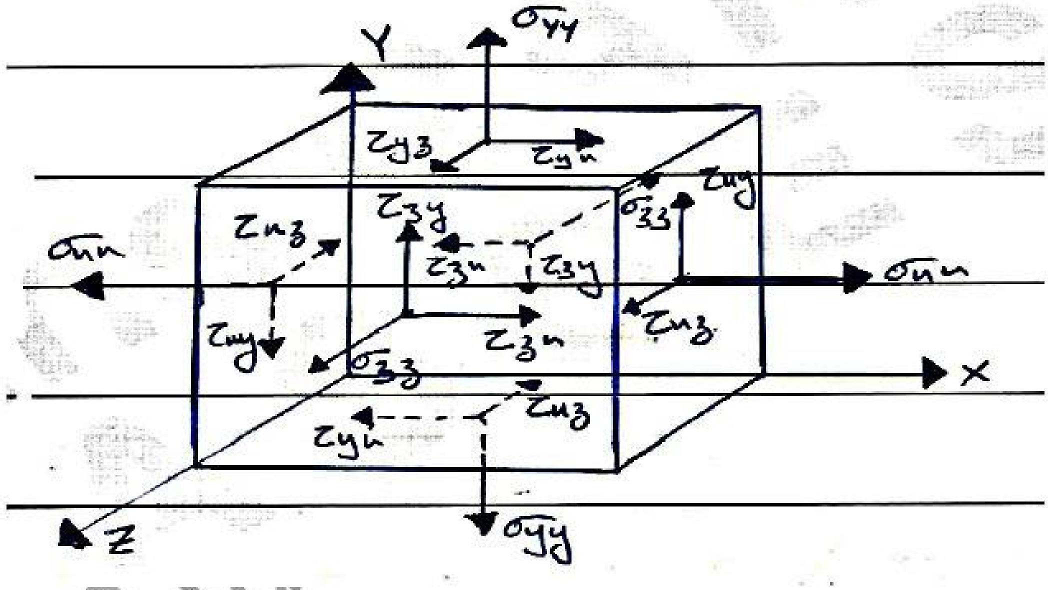
Strain energy

When a body structure undergoes deformation, strain energy is stored in the structure.
It is the summation of volumetric strain energy and shear strain energy.

$$U = U_v + U_s$$

<u>Strain energy density:</u>

Strain energy per unit volume is known as strain energy density.
Consider a 3D stress system as shown in the figure



Strain energy density due to normal stress

$\rightarrow u_v = \frac{1}{2}(\sigma_x \in_x + \sigma_y \in_y + \sigma_z \in_z)$

$\rightarrow u_v = \frac{1}{2E}[\sigma_x^2 + \sigma_y^2 + \sigma_z^2 - 2\vartheta(\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x)]$

$\rightarrow u_v = \frac{E}{2(1+\vartheta)(1-2\vartheta)}[(1-\vartheta).(\in_x^2 + \in_y^2 + \in_z^2) + 2\vartheta(\in_x\in_y + \in_y\in_z + \in_z\in_x)]$

Strain energy density due to shear stresses:

$\rightarrow u_s = \frac{1}{2}(\tau_{xy}\gamma_{xy} + \tau_{yz}\gamma_{yz} + \tau_{zx}\gamma_{zx})$

$\rightarrow u_s = \frac{G}{2}(\gamma_{xy}^2 + \gamma_{yz}^2 + \gamma_{zx}^2)$

$\Rightarrow$Strain energy = strain energy density×volume

Strain energy density due to hydrostatic loading

$u_v = \frac{3}{2E}[1-2\vartheta)\ p^2$

Here p is pressure loading

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Problem 1

A solid steel sphere (E = 210 GPa, ν = 0.3) is subjected to hydrostatic pressure p such that its volume is reduced by 0.4%.

(a) Calculate the pressure p.

(b) Calculate the volume modulus of elasticity K for the steel.

(c) Calculate the strain energy U stored in the sphere if its diameter is d = 150 mm.

(a) p = 700 MPa; (b) K = 175 GPa;
(c) U = 2470 J

Sol.

$E = 210$ GPa $\quad \nu = 0.3$

Hydrostatic Pressure. V_0 = Initial volume

$\Delta V = 0.004V_0$

Dilatation: $e = \dfrac{\Delta V}{V_0} = 0.004$

(a) Pressure

$$e = \frac{3\sigma_0(1-2\nu)}{E}$$

or $$\sigma_0 = \frac{Ee}{3(1-2\nu)} = 700 \text{ MPa}$$

Pressure $p = \sigma_0 = 700$ MPa ←

(b) Volume modulus of elasticity

$$K = \frac{\sigma_0}{e} = \frac{700 \text{ MPa}}{0.004} = 175 \text{ GPa} \quad \longleftarrow$$

(c) Strain energy (d = diameter)

$d = 150$ mm $\quad r = 75$ mm

From Eq. (7-57b) with $\sigma_x = \sigma_y = \sigma_z = \sigma_0$:

$$u = \frac{3(1-2\nu)\sigma_0^2}{2E} = 1.40 \text{ MPa}$$

$$V_0 = \frac{4\pi r^3}{3} = 1767 \times 10^{-6} \text{ m}^3$$

$U = uV_0 = 2470$ N · m = 2470 J ←